Quiz 5: 16.2, 16.3

Show all work clearly. Name any theorems you use. (You may only use theorems from these sections)

(1) Given the vector field $\vec{F}(x,y) = < 4x+5y, 5x-y >$ and the path C from (0,0) to (1,1) along the curve.

$$\begin{cases} x = t \\ y = \sin\left(\frac{\pi t}{2}\right) \end{cases}$$

a) Find the potential function f(x,y) such that $\vec{F} = \vec{\nabla} f(x,y)$.

$\vec{\nabla} f = \vec{F} \Rightarrow \langle \frac{\partial f}{\partial x}, \frac{\partial f}{\partial y} \rangle = \langle 4x+5y, 5x-y \rangle$

$\frac{\partial f}{\partial x} = 4x+5y$

(integrate dx)

$f(x,y) = 2x^2 + 5xy + c(y)$

$\partial/\partial y$

$\frac{\partial f}{\partial y} = 5x + c'(y) = 5x - y$

$\Rightarrow c'(y) = -y$

$c(y) = -\frac{1}{2}y^2 + c$

$\boxed{f(x,y) = 2x^2 + 5xy - \frac{1}{2}y^2 + c}$

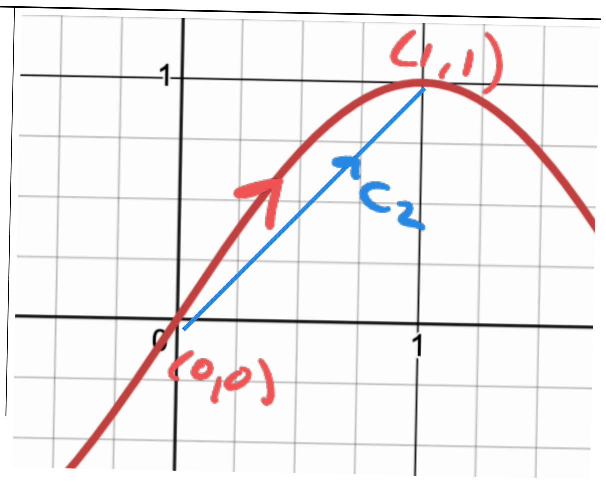
NOTE: worth taking time to check that $\vec{\nabla} f$ does equal $\vec{F}$

b) Find $\int_C \vec{F} \bullet d\vec{r}$ using two different methods. Explain

① Using Fundamental Theorem

$\int_C \vec{F} \cdot d\vec{r} = f(1,1) - f(0,0)$

$= \boxed{\frac{13}{2}}$



easier

② Since $\vec{F}$ is conservative, we can use a simpler path.

C_2: line segment (0,0)(1,1)

$x = t$
$y = t$ $\quad \vec{r} = \langle t, t \rangle \quad 0 \le t \le 1$

$\vec{F} = \langle 4t+5t, 5t-t \rangle = \langle 9t, 4t \rangle$

$\vec{r}\,' = \langle 1, 1 \rangle$

$\vec{F} \cdot \vec{r}\,' = 13t$

$\int_{C_2} \vec{F} \cdot d\vec{r} = \int_0^1 13t\,dt = \boxed{\frac{13}{2}}$

Answers match

③ Directly (much more work)

$\vec{F} = \langle 4t + 5\sin\frac{\pi t}{2}, 5t - \sin\frac{\pi t}{2} \rangle$

$\vec{r}\,' = \langle 1, \frac{\pi}{2}\cos\frac{\pi t}{2} \rangle$

$\vec{F} \cdot \vec{r}\,' = 4t + 5\sin\frac{\pi t}{2} + 5t\frac{\pi}{2}\cos\frac{\pi t}{2} - \frac{\pi}{2}\cos\frac{\pi t}{2}\sin\frac{\pi t}{2}$

$\int_C \vec{F} \cdot d\vec{r} = \int_0^1 \left(4t + 5\sin\frac{\pi t}{2} + 5t\frac{\pi}{2}\cos\frac{\pi t}{2} - \frac{\pi}{2}\cos\frac{\pi t}{2}\sin\frac{\pi t}{2}\right)dt$

very doable, but messy ... $\frac{13}{2}$

Note: On exam you will have to show all steps of integration, like what you used for u if you did a u-substitution